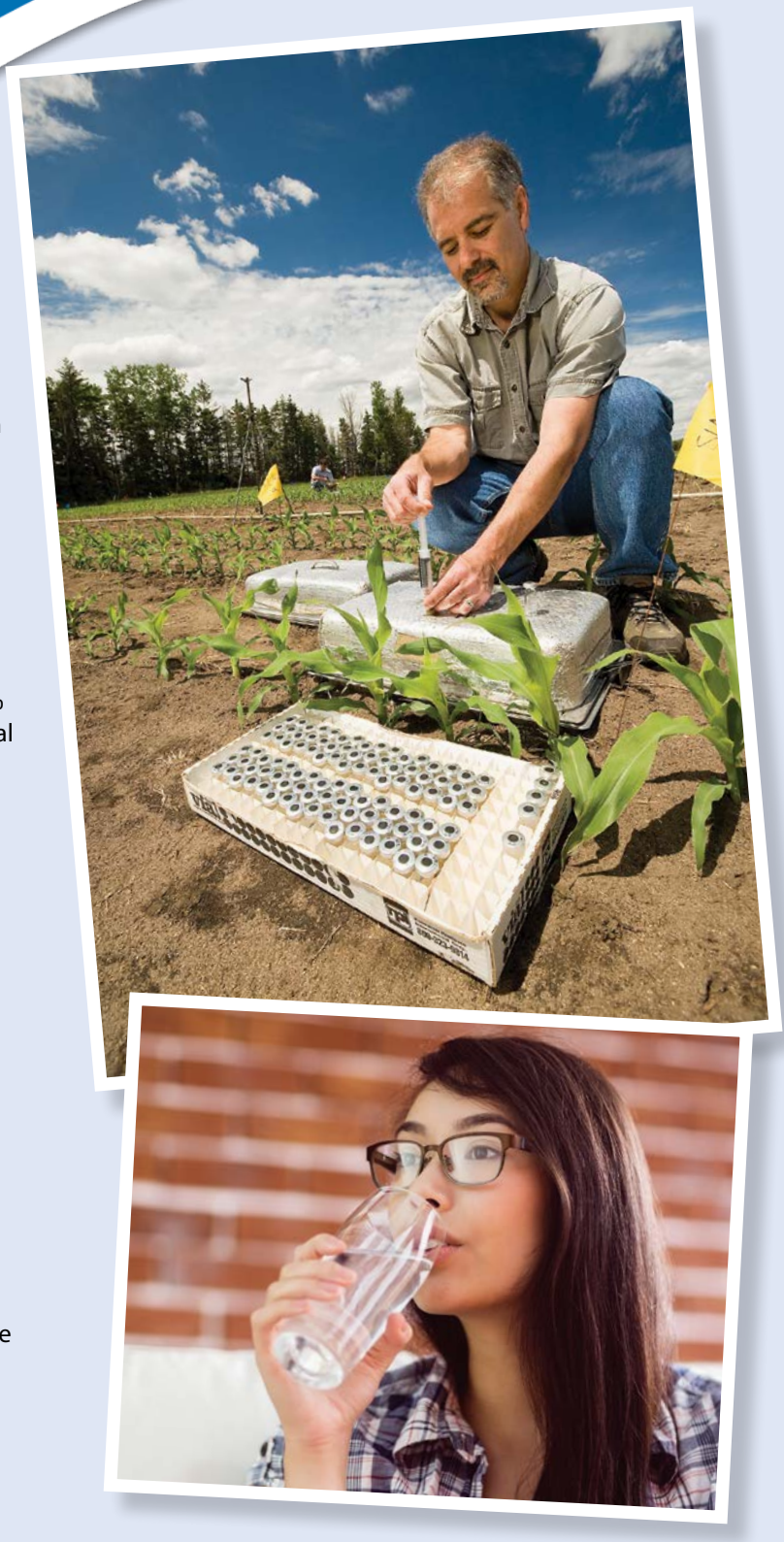
Chapter

8

Understanding Percent



Water is critical to the health of Earth and to your own health. About 71% of Earth's surface is covered by water. Of all the water on Earth, 97.5% is salt water, and the remaining 2.5% is fresh water. About 70% of the fresh water is permanently frozen. Only about 0.007% of all water on Earth is fresh water that is accessible for humans to drink.

Situations like this that involve very big or very small numbers often result in percents that are less than 1% or greater than 100%, or involve a fractional or decimal part. Environmental scientists often work with very small percents when they conduct research and present their findings to government officials or businesses. Environmental scientists analyze data collected through research to improve human and environmental health. For example, they may take water, soil, or air samples to study and develop plans to reduce pollution.

Big Idea

You can use percents to make comparisons between quantities. Percents are another way of using proportional reasoning.

Inquire and Explore

- How are decimals, fractions, and percents interrelated?
- How can you compare, represent, and communicate two quantities?

Get Ready

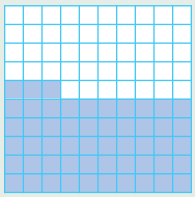
Percents

Percent means "out of 100."

$$\text{percent} = \frac{\text{part}}{\text{whole}}$$

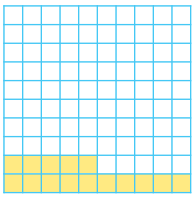
You can represent a percent by shading a hundred grid.

$$\begin{aligned}\%\text{ shaded} &= \frac{\textbf{part}\text{ of grid shaded}}{\text{one }\textbf{whole}\text{ grid}} \\ &= \frac{\textbf{53 squares}}{\textbf{100 squares}} \\ &= 53\%\end{aligned}$$



1. What percent of each grid is shaded?

a)



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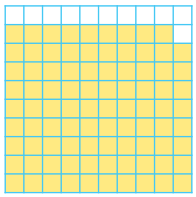

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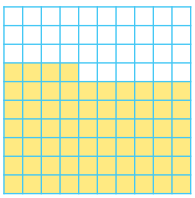
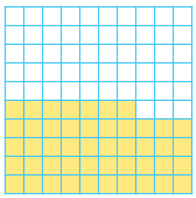


d)



2. Explain how you could use a hundred grid to express each percent.

a) 3%

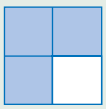
b) 46%

c) 97%

d) 15%

Fractions, Decimals, and Percents

The diagram represents $\frac{3}{4}$ because 3 out of 4 squares are shaded.



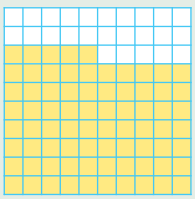
You can show $\frac{3}{4}$ as $\frac{75}{100}$.

$$\frac{3}{4} \xrightarrow{\times 25} \frac{75}{100}$$

You can write $\frac{75}{100}$ as a decimal, 0.75.

You can also write it as a percent, 75%.

To change a decimal to a percent, multiply by 100% and add a percent symbol.



3. Show each diagram as a fraction, a decimal, and a percent.

a) 

b) 

c) 

d) 

4. Draw a diagram to represent each number.

a) $\frac{3}{4}$ **b)** $\frac{1}{3}$

c) 0.1 **d)** 0.7

e) 5% **f)** 90%

Repeating Decimals

A **repeating decimal** has one or more digits that repeat over and over without ending.

$\frac{2}{3} = 0.\overline{6}$ C 2 ÷ 3 = **0.66666666...**

Some calculators display this result as 0.666666667. Why do you think this is?

A bar over the number shows the repeating part.

If more than one number repeats, the bar extends over the repeating numbers until the pattern repeats.

$1.1717171717\ldots = 1.\overline{17}$

5. Show as repeating decimals.

a) 0.3333333…

b) 0.4545454…

c) 0.1327272…

6. Show each fraction as a repeating decimal.

a) $\frac{9}{11}$ **b)** $\frac{7}{9}$

c) $\frac{5}{6}$ **d)** $\frac{4}{11}$

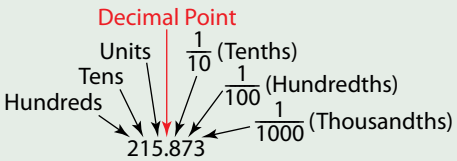
Estimating Percents

Place value tells you the value of a digit in a number.

To estimate the percent of a number, you can use percents you know.

- 52% of 250 is about 50% of 250.
- 50% of 250 is half of 250, or 125.

- 12% of 60 is about 10% of 60.
- 10% is about one tenth of 60, or 6.



7. State the place value of the 5 in each number.

a) 350.32 **b)** 28.5

c) 15.1 **d)** 1 698.75

e) 90.005 **f)** 498.089 5

8. Estimate each percent of a number. Show your thinking.

a) 22% of 85 **b)** 48% of 102

c) 75% of 70 **d)** 82% of 91

8.1

Understanding Large and Small Percents

Focus On…

In this lesson, I will learn to

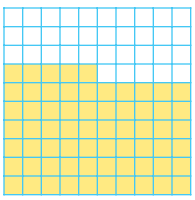
- visualize, describe, and identify situations where percents
  - are less than 1%
  - are greater than 100%
  - include a decimal or fractional portion

Explore and Analyze

Well over 7 billion people live on Earth today, and it is estimated that the world population currently increases by over 80 million people every year. That is well over 200 000 people per day, or 140 people every minute! It is hard to imagine just how big numbers like that really are, or just how small a part of that number a single person represents.

percent

- out of 100
- another name for hundredths
- for example, 65% means 65 out of 100 or $\frac{65}{100}$ or 0.65



Part A: What Percent of the World's Population Are You?

1. What **percent** do you make up of all the people in your

a) family? **b)** class? **c)** school?

2. Ask your teacher or research to find an approximate number for each of the following populations. Then, work with a partner to determine the approximate percent that one person represents compared with all the people in

a) your town/city/area **b)** your province

c) Canada **d)** the world

3. a) In #1 and #2, were any percents less than 1%? If so, which one(s)?

b) Were any of them **fractional percents**? If so, which one(s)?

fractional percent

- a percent that is not a whole number, but instead includes a fractional or decimal portion of a percent
- for example, 4.5%, $7\frac{3}{8}$%, 125.2%, $\frac{1}{2}$%, 0.42%

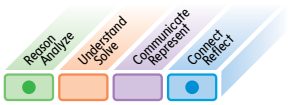
Part B: How Full Can You Make It?

4. Determine the maximum volume capacity of a measuring container.

5. Use a dry ingredient to fill the measuring cup as far beyond its capacity as you can (but try not to make a mess!). What volume is the container now holding?

6. What percent is the overfilled amount compared with the capacity shown on the container?

7. Discuss your findings from Parts A and B with a partner or group. Write about what you have learned about very large and very small percents. Consider these questions:



a) How can you use the words *part* and *whole* to explain how percents can be greater than 100 or less than 1?

b) What other situations would result in percents that are

i) greater than 100?

ii) less than 1?

iii) fractional percents?

Develop Understanding

Example 1: Represent Percents Visually Using Hundred Grids

For each situation,

- represent the percent visually using hundred grids
- describe the relationship between *the part* and *the whole*.

a) One serving of orange juice contains 135% of the recommended daily intake of vitamin C.

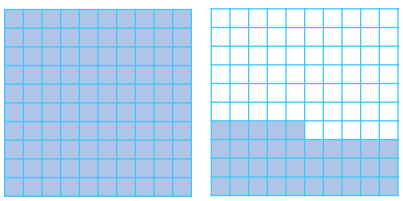
b) In the document called the United Nations Millennium Project, rich countries have agreed to try to contribute 0.7% of their yearly GDP (Gross Domestic Product, the total value of all goods and services produced) to help developing countries.

c) A credit card company charges an interest rate of $18\frac{3}{4}\%$ on unpaid balances.

Solution

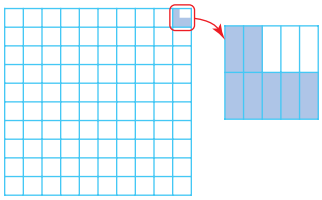
a) Since 135% is greater than 100%, you need more than one hundred grid.

You can represent 135% by shading one whole grid and 35 squares of a second grid.



The shading visually represents the vitamin C content of a serving of orange juice. The amount of vitamin C in one serving (the shaded parts of the two grids) is greater than the recommended daily amount (one whole grid) that it is being compared with.

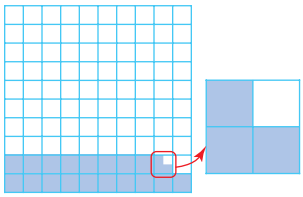
b) 0.7% is a fractional percent less than 1%. Zoom in on one square, or 1% of a hundred grid. Since 0.7% is the same as $\frac{7}{10}$%, divide the single square into ten equal sections and shade seven of the ten sections.



The shading visually represents the 0.7% goal of the UN's Millennium Project. The amount contributed by richer countries (the shaded part) is far less than their entire GDP, which is one whole grid.

c) $18\frac{3}{4}$% is a fractional percent between 1% and 100%, so you can use a single hundred grid.

Shade 18 squares to represent 18%, and shade $\frac{3}{4}$ of another square to represent $\frac{3}{4}$%.

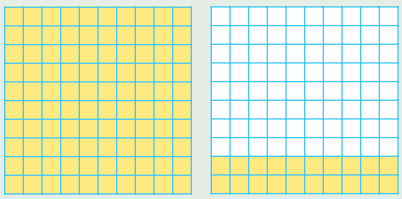


The shading visually represents the $18\frac{3}{4}$% interest rate. The interest charged by the credit card company (the shaded part) is less than the total balance on which the interest is being charged (one whole grid).

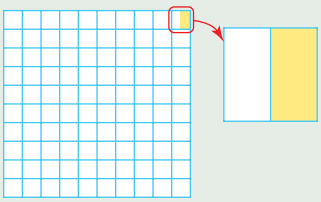
Show You Know

What percent does each diagram represent if a single hundred grid is 100%?

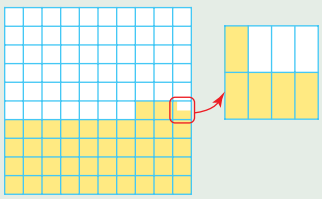
a)



b)



c)

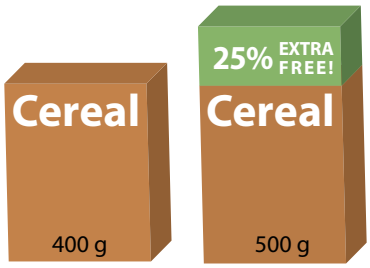


Example 2: Analyze Percents Greater Than 100%

Erin sees these two cereal boxes on a shelf while shopping for groceries with her family.

a) The new, bigger box is what percent of the original one?

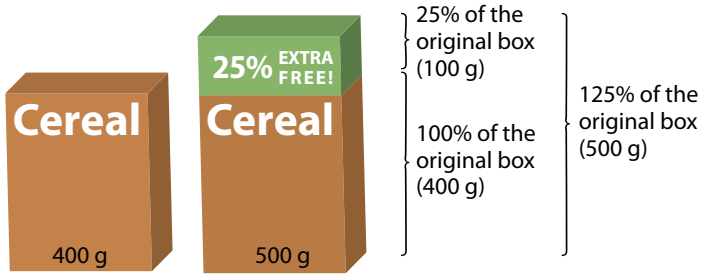
b) Erin suspects that the company has not accurately represented the size of the bonus amount on the outside of the new, bigger box. Based on the size of the boxes, is she right?



Solution

a) The new, bigger box is 500 g instead of the original 400 g. The new box contains 100 g more cereal than the original box. The bonus portion of the bigger box says "25% bonus" because 100 g is one quarter, or 25%, of the original 400-g box.

The new, bigger box is a combination of 100% of the original box plus 25% of the original box.



The bigger box is 125% of the original box. It is a percent greater than 100 because the *part* (the new box) is bigger than the *whole* (the original box).

b) If the box has been represented correctly, the green region should be 25%, or $\frac{1}{4}$ of the height of the original box. Measuring the heights of the images, the bonus part is actually closer to $\frac{1}{3}$, or approximately 33.3% of the height of the original box. The company has visually exaggerated the size of the bonus portion of the new box.

Show You Know

Ten years ago, a painting was estimated to be worth $1000. Today the value is $1600.

a) What percent does the painting's current value represent compared with the value ten years ago?

b) By what percent has the value increased over the value ten years ago?

Example 3: Analyze Percents Less Than 1% and Fractional Percents

The table shows population and land area information for Canada's provinces and territories.

a) Estimate which provinces and territories each make up less than 1% of Canada in both population and land area.

b) Of the ten provinces, PEI is the smallest by land area and by population, and BC is one of the largest. Use a calculator to determine the approximate percent of Canada's population and land area that each one represents.

	2015 Population	Land area* (km^2)
Canada	35 851 800	9 093 507
Newfoundland and Labrador	527 800	373 872
Prince Edward Island	146 400	5 660
Nova Scotia	943 000	53 338
New Brunswick	753 900	71 450
Québec	8 263 600	1 365 128
Ontario	13 792 100	917 741
Manitoba	1 293 400	553 556
Saskatchewan	1 133 600	591 670
Alberta	4 196 500	642 317
British Columbia	4 683 100	925 186
Yukon	37 400	474 391
Northwest Territories	44 100	1 183 085
Nunavut	36 900	1 936 113
*excluding freshwater lakes and rivers		

Solution

a) Canada's population is approximately 36 000 000 people. This represents 100%, so 1% would be approximately $\frac{1}{100}$ of 36 000 000:

if $100\% \approx 36\,000\,000$ people
then $1\% \approx 360\,000$ people

So, Prince Edward Island, Yukon, Northwest Territories, and Nunavut each represent less than 1% of Canada's total population.

Canada's total land area is approximately 9 000 000 km^2. This represents 100%, so 1% would be approximately $\frac{1}{100}$ of 9 000 000:

if $100\% \approx 9\,000\,000\ km^2$
then $1\% \approx 90\,000\ km^2$

So, Prince Edward Island, Nova Scotia, and New Brunswick each represent less than 1% of Canada's total land area.

b) British Columbia:

$$\text{\% land area} = \frac{\text{BC's } \textbf{part} \text{ of land area}}{\text{Canada's } \textbf{whole} \text{ land area}}$$
$$= \frac{\textbf{925 186 km}^2}{\textbf{9 093 507 km}^2}$$
$$\approx 0.102 \times 100\%$$
$$\approx 10.2\%$$

$$\text{\% population} = \frac{\text{BC's } \textbf{part} \text{ of population}}{\text{Canada's } \textbf{whole} \text{ population}}$$
$$= \frac{\textbf{4 683 100 people}}{\textbf{35 851 800 people}}$$
$$\approx 0.131 \times 100\%$$
$$\approx 13.1\%$$

Both of these are fractional percents because they have a whole number part and a decimal part. BC's land area represents slightly more than 10% of Canada's land area, and its population represents slightly more than 13% of Canada's population.

Prince Edward Island:

$$\text{\% land area} = \frac{\text{PEI's } \textbf{part} \text{ of land area}}{\text{Canada's } \textbf{whole} \text{ land area}}$$
$$= \frac{\textbf{5 660 km}^2}{\textbf{9 093 507 km}^2}$$
$$\approx 0.0006 \times 100\%$$
$$\approx 0.06\%$$

$$\text{\% population} = \frac{\text{PEI's } \textbf{part} \text{ of population}}{\text{Canada's } \textbf{whole} \text{ population}}$$
$$= \frac{\textbf{146 400 people}}{\textbf{35 851 800 people}}$$
$$\approx 0.0041 \times 100\%$$
$$\approx 0.41\%$$

Both of these are fractional percents less than 1%, because they consist only of a decimal part. PEI's population represents a very small part of Canada—less than 0.5% (half of 1%) of Canada's population. Its land area represents a very, very small part of Canada—much less than 1%. It is even less than 0.1% (one tenth of 1%) of Canada's land area.

Show You Know

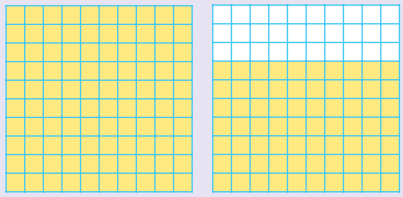
a) Estimate whether Alberta makes up greater than or less than 10% of Canada's population and of its land area.

b) Approximately what percent of Canada's population and land area does Nunavut represent?

Connect and Reflect

Key Ideas

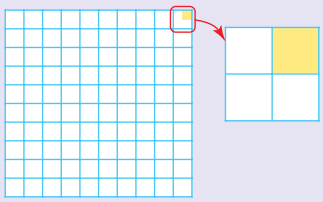
- Percents greater than 100% occur when the *part* is bigger than the *whole* that it is being compared with.



170%

The shaded *part* is more than one *whole* grid.

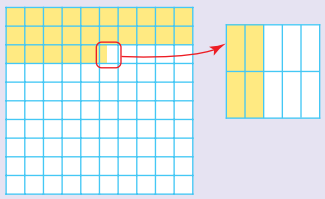
- Percents less than 1% occur when the *part* is significantly less than the *whole* that it is being compared with. Percents less than 1% are always fractional percents.



$\frac{1}{4}$% or 0.25%

The shaded *part* is less than a single square.

- Fractional percents between 1% and 100% have a whole number part and a fractional or decimal part.



$25\frac{4}{10}$% or 25.4%

The shaded *part* has a whole number of squares and a portion of another square.

Practise

For help with #1 to #3, refer to Example 1 on pages 257–258.

1. Represent each percent on a grid.

a) 125%

b) $10\frac{1}{2}$%

c) 0.4%

d) 262%

e) $\frac{7}{8}$%

f) 45.6%

2. How many hundred grids do you need to show each of the following percents?

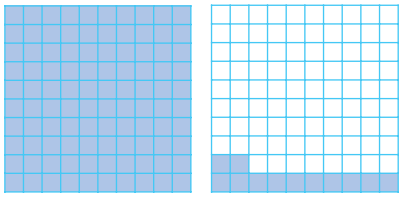
a) 300%

b) 466%

c) 1200%

3. One full grid represents 100%. What percent does each diagram represent?

a)



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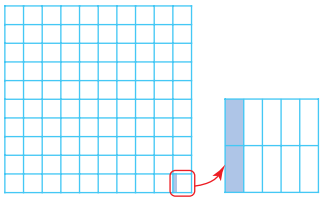
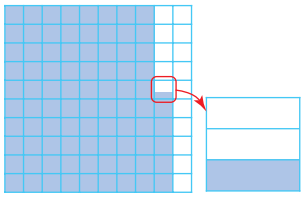
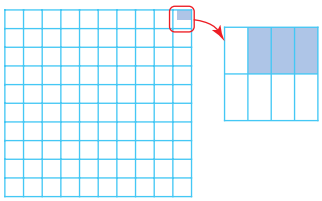


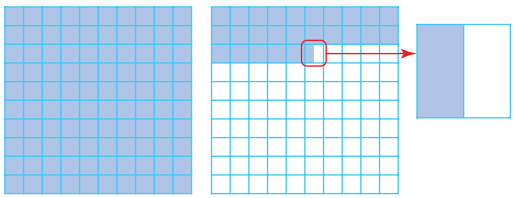
c)



d)



e)



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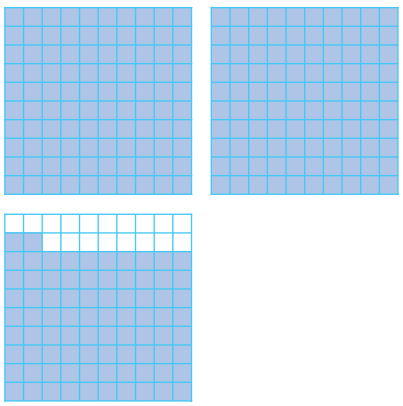
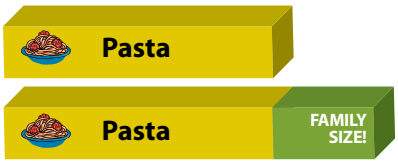


For help with #4 and #5, refer to Example 2 on page 259.

4. Two boxes of pasta are shown, one regular size and one family size.



a) Based on the size of the boxes, what percent is the family size package compared with the regular package?

b) What did you assume when you determined your answer to part a)?

c) If you were the package designer, how could you use percent to indicate the size of the bonus on the box?

5. The value per share of a certain stock was $60 last year. One year later, the value is $85.

a) By what percent has the value increased over the past year?

b) What percent is this year's value compared with last year's value?

For help with #6, refer to Example 3 on pages 260–261.

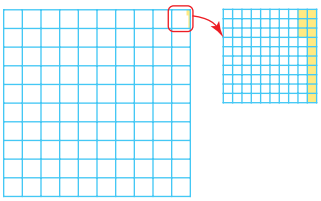
6. A census of Canadian agriculture found that there were 229 373 farms in Canada. Of these, 14 651 were dairy farms and 735 were greenhouse vegetable farms.

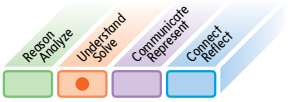
a) Estimate if either or both of these represent less than 1% of all farms in Canada. Explain your thinking.

b) Determine the approximate percent that each type of farm represents of all farms in Canada. Were your estimates in part a) accurate?

Apply

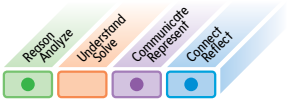
7. What percent does the diagram represent? Explain your answer.





8. The Kermode or Spirit bear is a rare white variety of the North American black bear found in coastal BC. It is BC's provincial mammal and holds a prominent place in local First Nations oral traditions. There are only about 400 Kermode bears out of a total of about 120 000 to 160 000 black bears in the province. According to these estimates, what is the maximum percent of Kermode bears in BC? What is the minimum?

9. A common phrase used in sports is "We've gotta give it 110% to win." What might this mean? Give 110% of what? Discuss with a partner and then explain your thoughts.



10. Provinces usually have laws about the ratio of adults to children at daycare facilities. Describe whether it would be possible for the adult-to-child ratio in this situation to be

a) less than 1% **b)** a fractional percent **c)** greater than 100%

11. **a)** If 8 represents 1%, what is the whole?

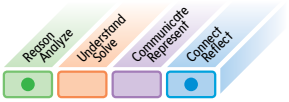
b) If 12 represents 25.5%, estimate the whole.

12. ✓ **Competency Check** Describe a real-world situation involving a percent that is

a) greater than 100%

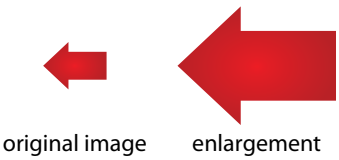
b) less than 1%

c) a fractional percent between 1% and 100%



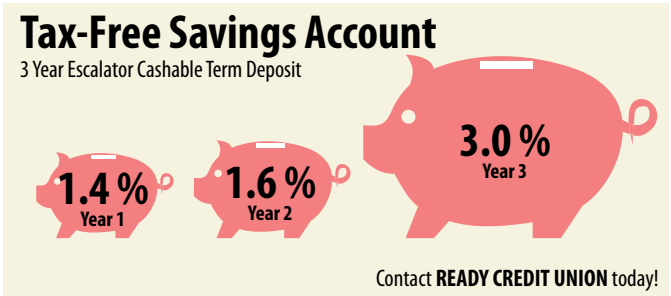
13. Why might a scientist studying water pollution use percents less than 1?

14. Heather used a copier to make the following enlarged image. The original and the copied image are shown. She was hoping to make a 250% enlargement. Did she succeed? Justify your reasoning.



original image enlargement

15. **Competency Check** A bank advertises a special savings account on its website. The interest rate on the account increases over a three-year period.



a) Use hundred grids to represent the three interest rates.

b) Which visual do you think does a better job of accurately representing the three rates: the bank's visual or the hundred grids? Explain.

c) The bank's visual makes it look like you would double your money after three years. Do you think this is true? Assume that you put $100 into this account. Estimate to support your answer.

16. The dimensions of several standard mattress sizes are shown. If the area of a single mattress is considered one whole, how can you use percent to express what the other three sizes represent in comparison?

Mattress Size Chart

Extend

17. How might you use hundred grid(s) to represent a very, very small percent, such as 0.000 001%? Where might you encounter a percent that is this small?

18. Describe a situation that would involve a percent greater than 1000%. Explain why the percent would be that large.

19. If a large square is divided into 8 smaller pieces, how could you represent $87\frac{1}{2}$% and $56\frac{1}{4}$%?

8.2

Fractions, Decimals, and Percents

Focus On…

In this lesson, I will learn to

- express and make connections among representations of percents, fractions, and decimals, including percents less than 1 and greater than 100, and fractional percents

Explore and Analyze

The Rockets goalie had a good start to the game, stopping **9 of the 10** shots faced. The second period saw **160% as many** shots as the first, yet an amazing **0.9375** save percentage performance held the Rockets in the game. Unfortunately, things took a turn for the worse in the third period. The Rockets goalie allowed **twice as many** goals despite facing only **one quarter** of the shots as the second period for a dismal **0.500** third period save percentage.

Percents, decimals, and fractions are different ways of showing the same value. Sports commentators often use statistics to report on the performance of athletes. This commentator is talking about the performance of a goalie using the statistics shown in the table.

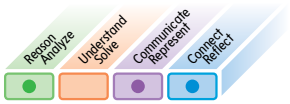
Goaltender Statistics			
Period	**Shots on Goal**	**Saves**	**Goals Against**
1	10	9	1
2	16	15	1
3	4	2	2
TOTAL	30	27	3

1. Change or rewrite each phrase that contains a **bold** statistic in the commentator's report so that it gives the same information but uses only

a) percents

b) fractions

c) decimals



2. Choose a single game or event in a sport or activity of your choice. You can research a real activity, or make up information for a fictitious activity. Write a commentary that includes at least two percents, two decimals, and two fractions to show some statistics about the team.

3. Share your report with a partner or group. Reflect on your discussion, and then write about presenting information in multiple ways using percents, decimals, and fractions. Consider these questions:

a) How did you decide which type of representation to use for each piece of information that you included in your report?

b) How are the decimal, percent, and fraction representations of a number the same? How are they different?

c) Which representations do you prefer to work with? Why?

Develop Understanding

Example 1: Convert Percents to Decimals and Fractions

Express 160%, 0.45%, and 12.6% as a

a) decimal **b)** fraction

Solution

a) Percent means "out of 100," or division by 100.

To write a percent as a decimal, remove the percent sign and divide by 100.

$$160\% = 160 \div 100$$
$$= 1.60 \text{ or } 1.6$$

What happens to the decimal point as you convert the percent to a decimal?

$$0.45\% = 0.45 \div 100$$
$$= 0.0045$$

$$12.6\% = 12.6 \div 100$$
$$= 0.126$$

Which digits of the decimal come from the whole number part of the percent? Which digits come from the decimal part of the percent?

b) To write a percent as a fraction, write the percent as a fraction out of 100. Multiply the numerator and denominator by the same number to eliminate decimals. Reduce to lowest terms if desired.

$$160\% = \frac{160}{100} \text{ or } \frac{8}{5} \text{ or } 1\frac{3}{5}$$

$$0.45\% = \frac{45}{100}$$
$$= \frac{0.45}{100} \times \frac{100}{100}$$
$$= \frac{45}{10\,000} \text{ or } \frac{9}{2000}$$

$$12.6\% = \frac{12.6}{100}$$
$$= \frac{12.6}{100} \times \frac{10}{10}$$
$$= \frac{126}{1000} \text{ or } \frac{63}{500}$$

Show You Know

Convert 225%, 0.7%, and 8.25% to a

a) decimal

b) fraction

Example 2: Convert Decimals to Percents and Fractions

Express 3.26, 0.125, and 0.0032 as a

a) percent

b) fraction

Solution

a) Percents represent the number of hundredths, or the first two places after the decimal point. To write a decimal as a percent, multiply by 100 and add a percent sign. When you multiply by 100, the decimal moves two places to the right.

$$3.26 = 326\%$$

$$0.125 = 12.5\%$$

$$0.0032 = 0.32\%$$

b) Decimals represent fractions out of 10, 100, 1000, and so on. To write a decimal as a fraction, use the place value and digits in the decimal to write a fraction with the appropriate denominator. Then reduce to lowest terms if desired.

$$3.26 = \frac{326}{100}$$
$$= \frac{163}{50}$$
$$= 3\frac{13}{50}$$

$$0.125 = \frac{125}{1000}$$
$$= \frac{1}{8}$$

$$0.0032 = \frac{32}{10\,000}$$
$$= \frac{2}{625}$$

How do you decide what denominator to use when writing a decimal as a fraction?

Show You Know

Convert 0.0004, 0.265, and 2.84 to a

a) percent

b) fraction

Example 3: Convert Fractions to Decimals and Percents

Express $\frac{1}{20}$, $\frac{71}{200}$, and $\frac{4}{3}$ as decimals and percents.

Solution

Method 1: Use an Equivalent Fraction Out of 100

Determine a multiplier or divisor that allows you to write an equivalent fraction out of 100.

The numerator of that fraction represents the percent.

Then, remove the percent sign and divide by 100 to get a decimal.

$$\frac{1}{20} \overset{\times 5}{=} \frac{x}{100} \overset{\times 5}{}$$
$$= \frac{5}{100}$$
$$= 5\%$$
$$= 0.05$$

$$\frac{71}{200} \overset{\div 2}{=} \frac{x}{100} \overset{\div 2}{}$$
$$= \frac{35.5}{100}$$
$$= 35.5\%$$
$$= 0.355$$

$$\frac{4}{3} \overset{\times 33.\overline{3}}{=} \frac{x}{100} \overset{\times 33.\overline{3}}{}$$
$$= \frac{133.\overline{3}}{100}$$
$$= 133.\overline{3}\% \text{ or } 133\tfrac{1}{3}\%$$
$$= 1.33\overline{3} \text{ or } 1.\overline{3}$$

How do you know that $133.\overline{3}\%$ is the same as $133\frac{1}{3}\%$? Which way do you prefer to express this percent?

Method 2: Divide to Get a Decimal First

You can think of a fraction as a division statement.

To convert a fraction to a decimal, use a calculator to divide the numerator by the denominator. Then, multiply by 100 and add a percent sign.

$$\frac{1}{20} = 1 \div 20$$
$$= 0.05 \times 100\%$$
$$= 5\%$$

$$\frac{71}{200} = 71 \div 200$$
$$= 0.355 \times 100\%$$
$$= 35.5\%$$

$$\frac{4}{3} = 4 \div 3$$
$$= 1.\overline{3}$$
$$= 1.33\overline{3} \times 100\%$$
$$= 133.\overline{3}\% \text{ or } 133\tfrac{1}{3}\%$$

Why can you write $1.\overline{3}$ as $1.33\overline{3}$? Why is it helpful when getting the percent?

Show You Know

Convert the fractions $\frac{3}{40}$, $\frac{171}{300}$, and $\frac{22}{15}$ to decimals and percents.

Example 4: Convert Percents With a Fractional Part

Nina noticed that the interest rate for a savings accounts is $1\frac{1}{2}\%$ at one bank and $1\frac{3}{4}\%$ at another bank. How can she convert these values to use them on her calculator?

Solution

It is easier to enter decimals into a calculator than fractions. First, write the values as percents with decimal parts instead of fractional parts. Then, remove the percent sign and use a calculator to divide by 100 to get a decimal equivalent for each.

$$\begin{aligned} 1\tfrac{1}{2}\% &= 1\% + \tfrac{1}{2}\% \\ &= 1\% + 0.5\% \\ &= 1.5\% \\ &= 1.5 \div 100 \\ &= 0.015 \end{aligned}$$

$$\begin{aligned} 1\tfrac{3}{4}\% &= 1\% + \tfrac{3}{4}\% \\ &= 1\% + 0.75\% \\ &= 1.75\% \\ &= 1.75 \div 100 \\ &= 0.0175 \end{aligned}$$

How can you turn the $\frac{3}{4}$ fractional part of the percent into a decimal part if you don't know the fraction?

Which digits of the decimal come from the whole number part of the percent? Which digits come from the fractional part?

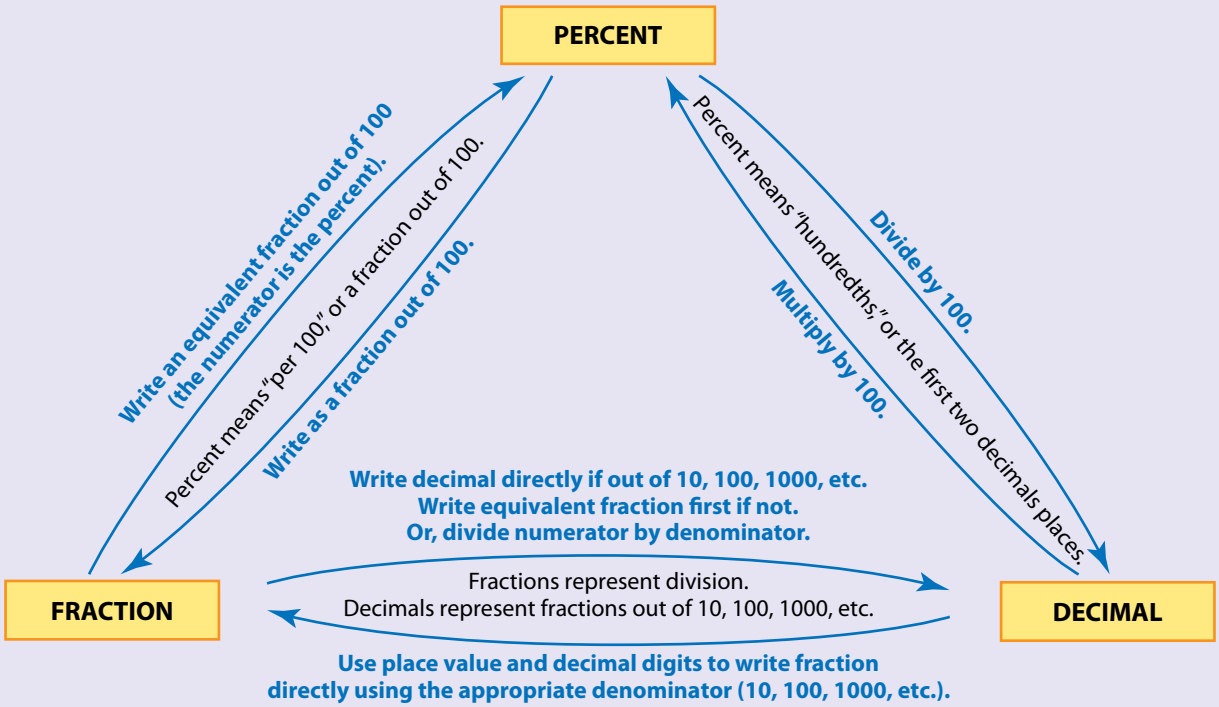
Show You Know

Express $3\frac{1}{4}\%$ and $20\frac{1}{2}\%$ as decimal values.

Connect and Reflect

Key Ideas

- To convert values between percent, fraction, and decimal numbers, use the relationships between these different ways of expressing numbers.



Practise

For help with #1, refer to Example 1 on page 267.

1. Convert each percent to a decimal and a fraction.

a) 248% **b)** 0.56% **c)** 75.5%

d) 5.93% **e)** 550% **f)** 0.8%

For help with #2, refer to Example 2 on page 268.

2. Convert each decimal to a percent and a fraction.

a) 0.0072 **b)** 0.548 **c)** 3.45

d) 0.256 **e)** 0.0005 **f)** 6.5

For help with #3, refer to Example 3 on page 269.

3. Convert each fraction to a decimal and a percent.

a) $\frac{1}{250}$ **b)** $\frac{81}{200}$ **c)** $\frac{7}{5}$

d) $\frac{51}{30}$ **e)** $\frac{123}{400}$ **f)** $\frac{3}{500}$

For help with #4, refer to Example 4 on page 270.

4. Express each percent as a decimal. Then write it as a fraction in lowest terms.

a) $5\frac{3}{4}\%$ **b)** $2\frac{7}{10}\%$

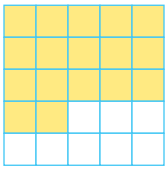
c) $21\frac{2}{5}\%$ **d)** $12\frac{2}{3}\%$

5. Copy and complete the table.

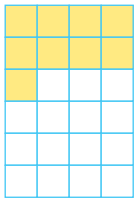
	Percent	Fraction	Decimal
	165%	$\frac{165}{100}$	1.65
a)	230%		
b)	0.38%		
c)	19.9%		

6. Express the shaded portion of each diagram as a fraction, a decimal, and a percent.

a)

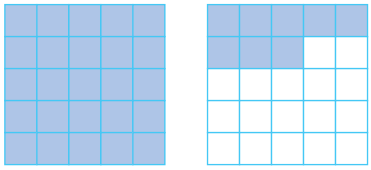


b)

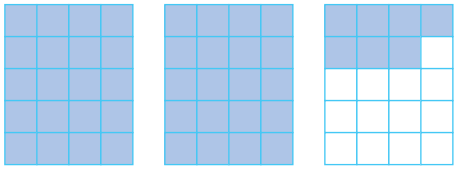


7. If one completely shaded grid represents one whole, write the shaded portion of each diagram as a fraction, a decimal, and a percent.

a)



b)



Apply

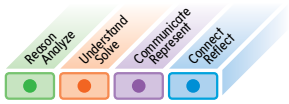
8. Which of the following values are equivalent? Explain your reasoning.

2.4 $\frac{12}{5}$ 250% $\frac{60}{25}$

9. How can you tell whether a value is more or less than one whole from the

a) fraction? **b)** decimal? **c)** percent?

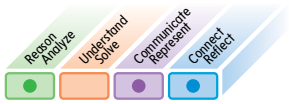
10. ✓ **Competency Check** A snack contains 0.9 g of fat. If Shaun consumes a total of 40 g of fat in one day, including the snack, what portion of Shaun's total fat consumption does the snack represent? Write your answer as a fraction, a decimal, and a percent.



11. Arrange the following numbers in ascending order.

145% $\frac{5}{8}$% 1.32 0.6% 0.65 33.5%

12. ✓ **Competency Check**



a) Copy the table below. Put "100%" partway down the percent column.

Percent	Decimal	Fraction
100%		

b) Enter values of your choice that are greater than 100% above it in the percent column. Enter values less than 100% below it.

c) Write the equivalent decimals and fractions for each entry in your percent column.

d) How do decimals for percents greater than 100% differ from those for percents less than 100%? How do the fractions differ?

13. A miner found 12 g of gold in a 2700-g sample of ore. What part of the sample is gold? Express your answer as a fraction and as a decimal. What percent of the sample is gold, to the nearest tenth of a percent?

14. A fundraising coordinator is creating an advertising flyer for an event. She wants to use either a fraction or a decimal number to represent each of the percents in the following statements. Decide whether a fraction or a decimal number is better and rewrite each statement using your chosen representation. Justify your choices.

a) Ticket sales are 130% of what they were at this time last year.

b) We are already at $60\frac{1}{2}$% of our target and we just started!

c) We have managed to cut our costs by 0.75%.

15. A fisheries worker recorded the following species and numbers of fish passing by a fish counter. Copy and complete the table.

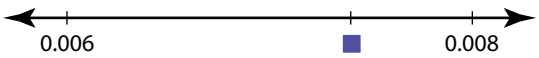
Species	Number	Percent of Total	Fraction of Total	Decimal Equivalent
Chinook	143			
Coho	122			
Steelhead	2			

16. Over five years, the circulation of a magazine increased from 25 000 copies to 150 000 copies per month. How does the new circulation compare to the circulation five years ago, expressed as a fraction, a decimal, and a percent?

17. Kim's resting heart rate is 75 beats per minute. A trainer advises Kim to have part of her workout at 90 beats per minute and part at 125 beats per minute, but not to exceed 150 beats per minute. Express each heart rate compared with the resting heart rate as a percent, a fraction, and a decimal.

Extend

18. Estimate the value of the indicated point on the line. Express the value as a fraction, a decimal, and a percent.



19. Identify at least three different fractions between $\frac{5}{7}$ and $\frac{6}{7}$. Write each fraction as a decimal and a percent.

8.3

Percent of a Number

Focus On...

In this lesson, I will learn to

- use mental math strategies to estimate the percent of a number
- use multiple approaches to solve problems involving percents less than 1 and greater than 100, including fractional percents

Explore and Analyze

Businesses often use percents to analyze sales and profits and to make comparisons and predictions. A school council is holding a car wash to raise money for school equipment. They are also planning to donate a portion of their profits to charity.

1. Last year the car wash raised about $5000. The fundraising committee sets a goal to increase their money raised to 115% of the amount raised last year.

a) Make a list of **percent benchmarks** by copying and completing the table.

percent benchmarks

- common percents that have a simple fractional equivalent
- percents that are convenient to work with using mental math

Percent Benchmarks			
$100\% = 1$ $10\% = \frac{1}{10}$ $1\% = \frac{1}{\square}$ $0.1\% = \frac{\square}{\square}$	$25\% = \frac{1}{4}$ $50\% = \frac{\square}{2}$ $75\% = \frac{\square}{\square}$	$12\frac{1}{2}\% = \frac{1}{8}$ $20\% = \frac{1}{\square}$	$33\frac{1}{3}\% = \frac{\square}{\square}$ $66\frac{2}{3}\% = \frac{\square}{\square}$

b) How can you use percent benchmarks to determine 115% of $5000 using mental math?

c) Last year, the fundraising team made a profit of $82\frac{1}{2}\%$. How might you determine a percent like $82\frac{1}{2}\%$ of $5000 using mental math?

d) Last year, the team donated 0.4% of the money raised to a charity. How would you determine 0.4% of $5000 using mental math?

2. Describe your personal strategies for determining the percent of a number using mental math. Compare the mental math strategies you used here to those of a partner, group, or your entire class.

Develop Understanding

Example 1: Use Mental Math to Find the Percent of a Number

Use mental math to determine each of the following.

a) 150% of \$5

b) 0.2% of \$1000

c) $1\frac{1}{2}$% of \$20 000

Solution

a) To determine 150% of \$5, find 100% and 50% separately, and then add.

100% of \$5 is \$5.00
50% of \$5 is \$2.50
So, 150% of \$5 is \$7.50

b) To determine 0.2% of \$1000, divide repeatedly by 10 to find 0.1%, and then multiply by 2.

100% of \$1000 is \$1 000
10% of \$1000 is \$100
1% of \$1000 is \$10
0.1% of \$1000 is \$1
So, 0.2% of \$1000 is \$2

c) ***Method 1: Determine 1% and $\frac{1}{2}$%, and Add***

100% of \$20 000 is \$20 000
10% of \$20 000 is \$2 000
1% of \$20 000 is \$200
$\frac{1}{2}$% of \$20 000 is \$100
So, $1\frac{1}{2}$% of \$20 000 is \$300

Method 2: Determine 3%, and Divide

100% of \$20 000 is \$20 000
10% of \$20 000 is \$2 000
1% of \$20 000 is \$200
3% of \$20 000 is \$600
So, $1\frac{1}{2}$% of \$20 000 is \$300

Show You Know

Use mental math to determine each of the following.

a) 350% of \$40

b) 0.6% of \$2000

c) 2.1% of \$30 000

Example 2: Calculate the Percent of a Number

A student council conducts an annual survey of students in their school. This year, they surveyed all 1600 students at school.

a) The survey showed that $\frac{1}{4}$% of students usually arrive more than an hour early for school. How many students does this represent?

b) Of the 680 students in grades 11 and 12, 92.5% have a cellphone. How many students does this represent?

c) The survey showed that the number of students who planned to walk or ride a bike to school this year is 108% of last year's number. If 875 students walked or rode a bike to school last year, how many plan to do so this year?

Solution

a) Multiply the *percent* by the *whole* school population to get the *part* that arrives early.

$$\begin{aligned}\text{Number Arriving More Than an Hour Early} &= \tfrac{1}{4}\% \text{ of all students} \\ &= 0.25\% \text{ of } \mathbf{1600} \\ &= 0.0025 \times 1600 \\ &= 4\end{aligned}$$

Why does it help to write a fractional percent like $\frac{1}{4}$% as a decimal percent first?

So, four students arrive more than an hour early for school.

How could you use mental math to determine your answer?

b) Multiply the percent by the whole grade 11 and 12 group to get the part that have a cellphone.

$$\begin{aligned}\text{Number With a Cellphone} &= 92.5\% \text{ of grade 11 and 12 students} \\ &= 0.925 \times \mathbf{680} \\ &= 629\end{aligned}$$

So, 629 students in grades 11 and 12 have a cellphone.

c) If there will be 108% as many students walking or riding a bike this year, multiply the number of students who did so last year by 108% to get the number for this year:

$$\begin{aligned}\text{Students Walking or Riding a Bike This Year} &= 108\% \text{ of students last year} \\ &= 1.08 \times \mathbf{875} \\ &= 945\end{aligned}$$

There will be 945 students walking or riding a bike this year.

Show You Know

a) If $62\frac{1}{2}\%$ of the 120 employees at a sporting goods company use public transit to get to work, how many people is this?

b) A company donates 0.8% of its profits to a children's charity each year. If their profits this year were \$1 200 000, how much will they donate?

c) A company produces 3200 of a particular item this year. Next year they plan to make 145% as many. How many will that be?

Connect and Reflect

Key Ideas

- You can use mental math strategies such as halving, doubling, and dividing by ten to find the percents of some numbers.
- To calculate the percent of a number, write the percent as a decimal and then multiply by the number.

Practise

For help with #1 to #3, refer to Example 1 on page 275.

1. Use mental math to determine each value.

a) 300% of 2000

b) $2\frac{1}{4}$ % of 60

c) 0.1% of 40

2. Use mental math to find the following.

a) 0.2% of \$60

b) 250% of \$400

c) $10\frac{1}{2}$ % of \$3100

3. The main elements that make up the human body are shown in the table. Use mental math to determine the amount of each element that is present in a 60-kg person.

Element	Percent of Body
Oxygen	65.0
Carbon	18.5
Hydrogen	9.5
Calcium	1.5
Phosphorus	1.0
Potassium	0.4
Sulfur	0.3
Sodium	0.2
Chlorine	0.2
Magnesium	0.1
Other elements	less than 1.0

For help with #4 to #6, refer to Example 2 on page 276.

4. Determine the percent of each number. Give your answer to the nearest hundredth.

a) 0.6% of 325

b) $15\frac{1}{4}$% of 950

c) 175% of $125.50

5. What is the percent of each number?

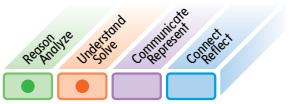
a) $\frac{5}{8}$% of 520

b) 75.4% of 200

c) 243% of $76.50

6. The manager of a store finds that, on average, 32.5% of customers bring their own bags. Determine how many people this might be on a day when 4000 customers visit the store.

Apply



7. Two hundred tickets are being sold for a school draw.

a) If you buy one ticket, what is your chance of winning, as a percent?

b) How many tickets would you need to purchase to have a 2.5% chance of winning?

8. The original price of a jacket was $86. A store manager marked the price down by 37.5%. By how much was the price reduced?

9. The highest point in Canada is Mount Logan, Yukon. Mount Logan is 159% as high as the highest point in Alberta, which is Mount Columbia. The elevation of Mount Columbia is 3747 m. What is the elevation of Mount Logan?

Mount Logan

10. The volume of solid ice is about 110% the volume of liquid water. What is the volume of ice created when 750 mL of water freezes?

11. A manufacturer of electric hybrid vehicles claims its vehicle will travel 200% as far on a full tank of gas as the regular model of the same vehicle. If the regular vehicle travels an average of 550 km on a full tank, how far will the hybrid go?

12. ✓ **Competency Check**

a) The expression "one in a million" means that a person is unique or very special. What percent does this represent?

b) If you were literally "one in a million," how could you use mental math to determine approximately how many other people are exactly like you in

i) all of Canada (population in 2016 is about 36 million)

ii) the entire world (population in 2016 is about 7.5 billion)

13. A real estate agent receives 6.5% commission on the first $100 000 of a house's selling price, and 3.5% on the remaining amount.

a) What does *commission* mean?

b) If a house sells for $845 000, how much commission does the real estate agent make?

14. Answer and explain your reasoning for the following statement: 12.5% of 500 is the same as 25% of what number?

Extend

15. Chelsea got a 6.4% bonus on her paycheque this month. If her total paycheque was $8171.52, what would her pay have been without the bonus?

16. If Laina earns 40% more than Caleb, what percent less does Caleb earn than Laina?

17. Ming took 16 shots and has a 37.5% scoring average after her first basketball game this season. She wants to raise her overall scoring average to 60% by the end of the next four games. If she takes an average of 24 shots each game, how many baskets does she need to make in the next four games to achieve this?

8.4

Combined Percents and Percent of a Percent

Focus On…

In this lesson, I will learn to

- use multiple strategies to solve problems involving combined percents and percent of a percent

Brackendale Eagles Provincial Park lies along the Squamish River north of Vancouver. It is one of the most significant winter locations for bald eagles in North America. Every year the number of eagles in the area is tracked at the Brackendale Winter Eagle Festival and Count. The number of eagles depends on the number of salmon in the river, which can change dramatically from year to year. In 2015, 637 eagles were counted, representing a 61% decrease from the 1617 birds counted in 2014. In 2016, the count was only 411, a 35% decrease from the 2015 number. What overall percent decrease does this represent over the two-year period?

Explore and Analyze

Obtain at least 40 small countable objects such as beads, tokens, linking cubes, or popcorn kernels. Work with a partner to investigate the following.

1. If you increase an amount by 50%, and then decrease the result by 50%, is the final amount the same as what you started with?

2. If you decrease an amount by 50%, and then decrease the result by 50%, is the final amount zero?

3. Choose percents other than 50% and investigate the results you get with various combinations of percent increases and/or decreases.

4. Discuss your findings with your partner or a group. Reflect on your discussion, and then write about what you have discovered when there are successive percent increases or decreases.

Develop Understanding

Example 1: Apply Percent of a Percent

Only about 29% of the total surface area of Earth is made up of land, and only about 38.4% of this land area is agricultural land. Approximately what percent of Earth's total surface area is agricultural land?

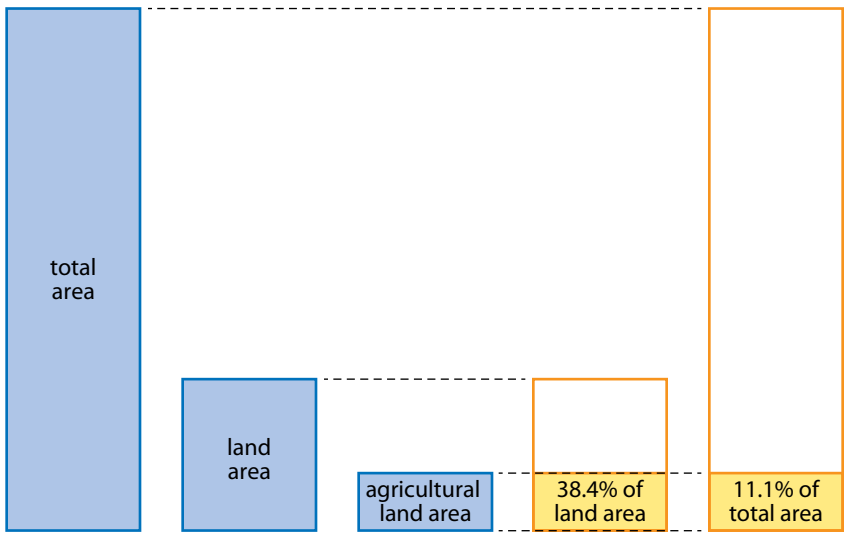
Solution

This situation involves a percent of another value that is also expressed as a percent of a total.

Think of the total surface area of Earth as 100%. Then, the percent of the total surface area that is agricultural land is 38.4% of land area. So,

38.4% of land area
= 38.4% of 29% of total surface area
= $0.384 \times \mathbf{0.29}$
= $0.111\ 36 \times 100\%$
≈ 11.1%

Approximately 11.1% of Earth's surface is agricultural land.



Show You Know

About 28.4% of all the agricultural land on Earth is arable land, which means it can be ploughed. What percent of all land area on Earth is arable land?

Example 2: Combine Percent Increases and Decreases

The number of students at a school is currently 1200. School officials predict that the number of students will increase by 9% next year, but then drop by 12% the following year. How many students will be at the school two years from now?

Solution

Method 1: Determine the Increase and Decrease Separately

$$\begin{aligned}\text{Increase Next Year} &= 9\% \text{ of students } \textit{this year}\\ &= 0.09 \times \mathbf{1200}\\ &= 108\end{aligned}$$

$$\begin{aligned}\text{Total Number Next Year} &= \text{number this year} + \text{increase}\\ &= \mathbf{1200} + \mathbf{108}\\ &= 1308\end{aligned}$$

$$\begin{aligned}\text{Decrease the Following Year} &= 12\% \text{ of students } \textit{next year}\\ &= 0.12 \times \mathbf{1308}\\ &\approx 157\end{aligned}$$

$$\begin{aligned}\text{Total Number the Following Year} &= \text{number next year} - \text{decrease}\\ &\approx \mathbf{1308} - \mathbf{157}\\ &\approx 1151\end{aligned}$$

Is adding 9% and then subtracting 12% the same as simply subtracting 3%? Are these two percents of the same value?

Method 2: Combine Percents First

An increase of 9% means that the number of students next year will be 100% + 9%, or 109%, of the number *this year*. Use this combined percent to determine the number next year directly in a single step.

$$\begin{aligned}\text{Total Number Next Year} &= 109\% \text{ of students } \textit{this year}\\ &= 1.09 \times \mathbf{1200}\\ &= 1308\end{aligned}$$

Why is it okay to add the 100% and 9% together? Are these two percents of the same value?

A decrease of 12% means that the number of students the following year will be 100% − 12%, or 88%, of the number *next year*. Use this combined percent to determine the number the following year directly in a single step.

$$\begin{aligned}\text{Total Number the Following Year} &= 88\% \text{ of students } \textit{next year}\\ &= 0.88 \times \mathbf{1308}\\ &\approx 1151\end{aligned}$$

Why is it okay to subtract 12% from 100% here? Are these two percents of the same value?

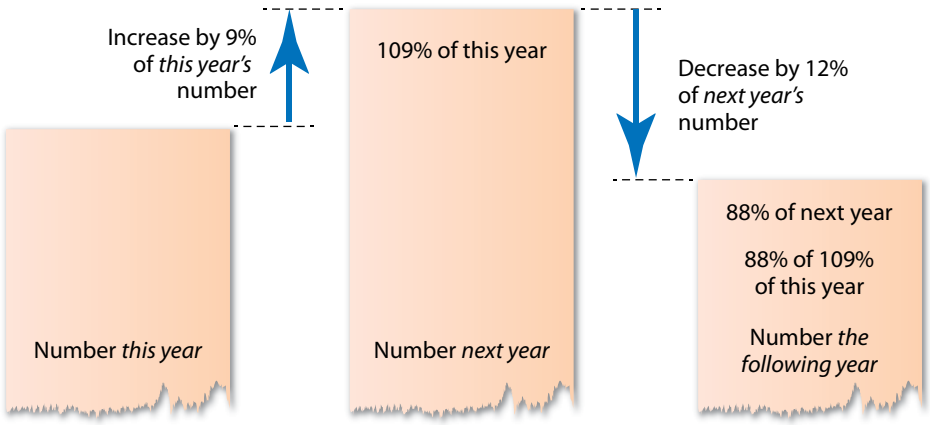
Method 3: Use Combined Percents and Percent of a Percent

An increase of 9% means that the number of students next year will be 100% + 9%, or 109%, of the number *this year*. A decrease of 12% means that the number the following year will be 100% − 12%, or 88%, of the number *next year*.

Use these two combined percents to determine the number of students the following year directly in a single step:

$$\begin{aligned}\text{Total Number the Following Year} &= 88\% \text{ of number } \textit{next year} \\ &= 88\% \text{ of } 109\% \text{ of number } \textit{this year} \\ &= 0.88 \times 1.09 \times \mathbf{1200} \\ &\approx 1151\end{aligned}$$

How does the number of separate calculations compare with the number in the other two methods?



Show You Know

A company is currently storing 20 000 items in a warehouse. Next month they expect a 15% increase in the number of items. The following month they expect that the number will increase by another 20%. How many items will be in the warehouse after the two months?

Connect and Reflect

Key Ideas

- You can combine percents by adding or subtracting if they are percents of the same value.
- You can multiply percents in situations involving a percent of another percent.
- You can use percents of percents to determine amounts that result from consecutive percent increases or decreases.

Practise

For help with #1 to #3, refer to Example 1 on page 281.

1. Determine each value.

a) 60% of 30%

b) 120% of 82%

c) 38.5% of 50%

d) 40% of 0.8%

2. About 2.5% of all the water on Earth is fresh water. About 70% of this fresh water is permanently frozen in glaciers. What percent of all the water on Earth is the fresh water in glaciers?

3. A school decided that they would spend 50% of their equipment budget this year on computer equipment, and that 12.5% of the money spent on computer equipment would be for new printers. What percent of the entire equipment budget will be spent on new printers?

For help with #4 and #5, refer to Example 2 on pages 282–283.

4. Determine each final value.

a) An initial value of 600 is first increased by 28%, and then the result is decreased by 15%.

b) An initial value of 1820 is first decreased by 45%, and then the result is increased by 31%.

c) An initial value of 80 is first increased by 6%, and then the result is increased by 20%.

d) An initial value of 24 000 is first decreased by 65%, and then the result is decreased by 52%.

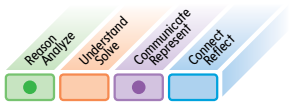
5. A herd of 100 caribou was moved to a new location. The population increased by 10% in the first year and then increased by 20% in the second year.

a) What is the population at the end of the second year?

b) Explain why there was not a 30% increase in population over the two years.

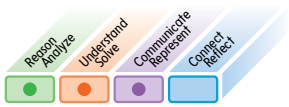
Apply

6. Choose a number. Draw a diagram to show visually the relationships between values and percents when your number is decreased by 40% and then increased by 25%.



7. ✓ **Competency Check** One year a family increases the size of their vegetable garden by 20%. A year later they decrease the size by 20%.

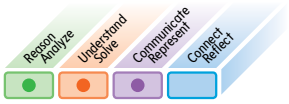
a) Does this mean that their garden is the same size as it was to start with? Justify your answer mathematically.



b) Would the final size of the garden be the same if the decrease happened before the increase? Show your thinking.

8. Kyle says that a population increase of 15% one year followed by an increase of 10% the next year is the same as a population increase of 25% over two years. Is Kyle correct? Explain your reasoning.

9. ✓ **Competency Check** The length of the Fraser River is about 1370 km, and the length of the Columbia River is about 2000 km. Copy the statements below and fill in the blanks.



a) The Columbia River is about ■% of the length of the Fraser River.

b) The Columbia River is about ■% longer than the Fraser River.

c) The Fraser River is about ■% of the length of the Columbia River.

d) The Fraser River is about ■% shorter than the Columbia River.

e) Use the words *part* and *whole* to explain why none of the percent you wrote in parts a) to d) are the same.

10. Determine a single percent increase or decrease that is equivalent to each of the following:

a) an increase of 10% followed by a decrease of 40%

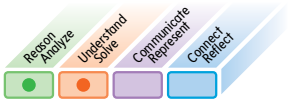
b) a decrease of 35% followed by an increase of 90%

c) a decrease of 30% followed by an increase of 12%

d) an increase of 75% followed by a decrease of 15%

11. Which is greater, 60% of 50% of 40%, or 50% of 50% of 50%? Justify your reasoning.

12. What single percent is equivalent to 130% of 120% of 110%?



Extend

13. If you increase a number by 62%, what percent decrease do you need to apply to the result to get back to your original number?

14. Samantha places a marker 12 m away from a door. She moves it toward the door so that after every minute that passes, she reduces the distance between the marker and the door by 50%.

a) Approximately how far away from the door is the marker after 10 minutes?

b) Will the marker ever reach the door? Explain your thinking.

8.5

Percent and Financial Literacy

Focus On…

In this lesson, I will learn to

- use a variety of strategies to solve problems involving multiple percents in financial situations

Explore and Analyze

Some provinces have separate PST (provincial sales tax) and GST (federal goods and services tax) that are charged on items purchased. Other provinces have a single tax called the HST (harmonized sales tax), which is a combination of provincial and federal taxes. For a three-year period, BC temporarily switched to a single 12% HST instead of having separate taxes of 5% GST and 7% PST.

Reason Analyze | Understand Solve | Communicate Represent | Connect Reflect

1. Is charging a separate 5% GST and 7% PST equivalent to charging a single 12% tax? Investigate using your own examples.

2. a) If an item was on sale for 40% off, but then was marked down by an "extra 25% off," how could you calculate the sale price? What single discount is this equivalent to?

b) How can you determine the total cost of an item if it is on sale for 40% off and is subject to 12% tax? What single discount would this be equivalent to?

3. Discuss your findings with a partner or group. Write about what you have learned about making calculations involving discounts and taxes. Consider the following in your reflection.

a) What different approaches can you take when working with combined discounts and taxes? Which methods do you find most efficient? Why?

b) What affects whether you can add or subtract percents prior to calculation?

Develop Understanding

Example 1: Determine the Total Cost of a Regular-Priced Item

What is the total cost of a remote control helicopter that is regularly priced at $160 if PST is 7% and GST is 5%?

Solution

Method 1: Determine the Taxes Separately

Calculate both taxes on the regular price of the helicopter.

GST = 5% of regular price
= 0.05 × **$160**
= $8.00

PST = 7% of regular price
= 0.07 × **$160**
= $11.20

Total Cost = regular price + GST + PST
= **$160** + **$8.00** + **$11.20**
= $179.20

Method 2: Combine the Taxes First

Since both taxes are percents of the same value, you can add the two percents first. When combined, the total tax will be 5% + 7%, or 12% of the regular price.

Total Tax = 12% of regular price
= 0.12 × **$160**
= $19.20

Total Cost = regular price + total tax
= **$160** + **$19.20**
= $179.20

Method 3: Combine the Cost and Tax Percents First

The regular price represents 100% and the taxes represent 12% of the regular price. When combined, the total cost will be 100% + 12%, or 112% of the regular price.

Total Cost = 112% of regular price
= 1.12 × **$160**
= $179.20

In the decimal value of 1.12, what part of what you pay does the 1 represent? What part does the .12 represent?

How does the number of separate calculations that you have to do compare among the three methods?

Show You Know

A pair of boots costs $65. Determine the total cost if GST is 5% and PST is 7%.

Example 2: Determine the Total Cost for a Sale Item

Ariana notices that a bike she wants to buy is on sale for 30% off. She also knows that in BC there is no PST charged on bikes, so she will only be charged 5% GST. If the regular price is $420, how much will her bike cost in total?

Solution

Method 1: Determine Sale Price First, and Then the Total Cost

Determine the discount amount, then the sale price, then the tax, and finally the total cost.

Discount = 30% of *regular price*
= 0.3 × **$420**
= $126

Sale Price = regular price − discount
= **$420** − **$126**
= $294

Tax = 5% of *sale price*
= 0.05 × **$294**
= $14.70

Total Cost = sale price + tax
= **$294** + **$14.70**
= $308.70

Why isn't subtracting 30% and then adding 5% here the same as simply subtracting 25%? Are these both percents of the same value?

Method 2: Use Combined Percents to Determine Sale Price and Total Cost

A discount of 30% means that the sale price will be 100% − 30%, or 70%, of the *regular price*.

Sale Price = 70% of *regular price*
= 0.7 × **$420**
= $294

A tax rate of 5% means that the total cost will be 100% + 5%, or 105%, of the *sale price*.

Total Cost = 105% of *sale price*
= 1.05 × **$294**
= $308.70

Method 3: Use Combined Percents and Percent of a Percent

A 30% discount means that the sale price will be 70% of the *regular price*. Adding 5% tax means that the total cost will be 105% of the *sale price*. So the total cost will be 105% of 70% of the regular price of the bike.

$$\begin{aligned}\text{Total Cost} &= 105\% \text{ of } \textit{sale price}\\ &= 105\% \text{ of } 70\% \text{ of } \textit{regular price}\\ &= 1.05 \times 0.7 \times \$420\\ &= \$308.70\end{aligned}$$

How does the number of separate calculations compare with the number of calculations in the other two methods?

Show You Know

Josh is buying a skateboard while shopping in Bellingham, Washington, where the sales tax rate is 8.7%. The regular price is $128, but it is on sale for 20% off. Determine the total cost of the skateboard.

Example 3: Determine Combined Percent Discounts

Nick wants to buy a gaming system. It usually sells for $400, but it is on sale at a store for 20% off. He finds an online coupon that will give him 40% off of any one item, including sale-priced items.

a) What is the final sale price of the system after the store's discount and the additional discount for the coupon?

b) What single overall percent discount is this combination of discounts equivalent to?

Solution

a) Use combined percents and percent of a percent to find the discounts directly. The store's discount of 20% means that the first sale price is 80% of the *regular price*. The coupon discount of 40% means that the final sale price is 60% of the *first sale price*.

$$\begin{aligned}\text{Final Sale Price} &= 60\% \text{ of } \textit{first sale price}\\ &= 60\% \text{ of } 80\% \text{ of } \textit{regular price}\\ &= 0.6 \times 0.8 \times \$400\\ &= \$192\end{aligned}$$

Why is 20% off followed by 40% off not the same as 60% off?

The final sale price is $192.

b) ***Method 1: Use Percent of a Percent***

Think of the regular price as 100%. Then use percent of a percent to determine the overall percent of the regular price that will be paid.

$$\begin{aligned}\text{Overall Percent of Regular Price Paid} &= 60\% \text{ of } 80\% \text{ of } \textit{regular price}\\ &= 0.6 \times 0.8 \times 100\%\\ &= 48\%\end{aligned}$$

The final sale price is 48% of the regular price. Therefore, the overall discount must be 100% − 48%, or 52% off of the regular price.

Method 2: Use the Total Cost as a Percent of Original Cost

Determine the percent paid for the gaming system. Then, subtract from 100%.

$$\text{Percent Paid} = \frac{\text{final sale price}}{\text{regular price}} \times 100\%$$
$$= \frac{192}{400} \times 100\%$$
$$= 48\%$$

$$\text{Overall Percent of Regular Price Paid} = 100\% - 48\%$$
$$= 52\%$$

Show You Know

Kayla is buying a craft kit that has a regular price of $89 but is on sale for 25% off. The store is holding a one-day sale in which the craft kit will be put on clearance for 60% off the already reduced price.

a) What is the final sale price of the craft kit?

b) What single overall percent discount is this combination of discounts equivalent to?

Connect and Reflect

Key Ideas

- You can calculate PST and GST separately or together by adding the tax percents first because they are both percents of the same number.
- You can determine sale prices in several steps by calculating the discount first and then subtracting, or in a single step by subtracting the discount percent from 100% and multiplying.
- You can determine the total cost in several steps by calculating the tax first and then adding, or in a single step by adding the tax percent to 100% and multiplying.
- In a situation involving consecutive percent discounts,
  - you *cannot* determine the overall discount by adding the two individual discount percents because they are not percents of the same number
  - you *can* determine the overall discount by multiplying (using percent of a percent)

Practise

For help with #1 to #3, refer to Example 1 on page 287.

1. Chris purchases the following items. Find the total cost of each item including 5% GST and 7% PST.

a) two binders at $4.99 each

b) one math set for $3.99

c) a backpack for $19.99

2. Copy and complete the table. Use 5% GST and 7% PST.

	Item Purchased	Price	Total Tax	Total Cost
a)	Boots	$119.99		
b)	Pants	$89.99		
c)	Gloves	$39.99		
d)	Helmet	$189.99		

3. Ravi downloads 3 movies at a cost of $19.99 each. What is the total cost of the movies, including 5% GST and 7% PST?

For help with #4 and #5, refer to Example 2 on pages 288–289.

4. A photo frame is regularly $36 but is on sale for 40% off. Determine the total cost including 5% GST and 7% PST.

5. What is the total cost of a $1300 patio set that is on sale for 25% off, including 5% GST and 7% PST?

For help with #6 and #7, refer to Example 3 on pages 289–290.

6. A store discounts items by 50% off the original price one week. The following week it takes an additional 10% off the already reduced price. The regular price of a solid state drive is $85.

a) What is the reduced price in the second week?

b) What single overall percent discount is this equivalent to?

7. Kim wants to buy a winter coat that regularly costs $120, but it is on sale for 25% off. Kim has a coupon that gives a discount of 30% off any item, including items already on sale.

a) What will Kim have to pay if she uses the coupon?

b) What single overall percent discount is this equivalent to?

Apply

8. Draw a diagram to show how to represent

a) the total cost of a $1000 item with and without tax

b) the sale price of a $400 item before and after being discounted

9. Arjay was thinking of buying a car worth $23 000, but delays purchasing the car for a year. During that year, the cost of the car increases by 3.2%.

a) What is the price of the car when Arjay purchases it?

b) What is the total cost of the car including 5% GST and 7% PST?

10. An item is discounted by 40% one day. The next day the reduced price is discounted by 60%. Does this mean that it is free? Explain your reasoning.

11. Callum needs to buy 4 tires that are regularly $145 each. They are on sale for 20% off. Callum still needs to pay 5% GST, 7% PST, and an after-tax environmental levy of $5 per tire. How much will the tires cost?

12. Four friends are eating dinner at a restaurant. The bill comes to $85 before tax and tip.

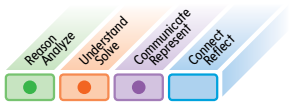
a) The restaurant adds 5% GST and the friends agree to add 20% tip on top of the tax. What is the total bill?

b) Since the friends had different meals, they decide to split the total bill as follows:

- one person pays 35%
- one person pays 25%
- two people pay 20% each

Determine how much each person spends on dinner.

13. ✓ **Competency Check** Sports R Us offers a 10% off discount one day and then an additional 10% off the sale price the next day. Sports Galore offers a 20% discount on one day only. Keifer wants to buy a new goalie mask that has a regular price of $200 at both stores. Which store gives the better buy? Explain your reasoning.



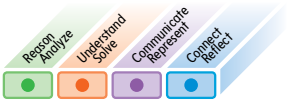
Extend

14. A ski jacket has been marked down on three occasions, first 20% off, then 25% off the new price, and finally 50% off the previous price. What is the overall percent saved?

15. An item is marked on sale for 30% off. What additional discount is necessary for the overall discount to be equivalent to 80%?

16. The selling price of a camera is 35% more than it costs the store to buy the camera from the manufacturer. It is on sale at a discount of 20% off the selling price. How much profit does the store still gain?

17. A sound system is auctioned on the Internet. The starting bid is $100. The second bid is 135% of the first bid. The third bid is 257% of the second bid. There are then 5 more bids, each 10.5% over the previous bid. The winning bid comes with only seconds left and is only 0.1% greater than the previous bid. What is the winning bid? What assumptions did you make to arrive at your answer?



18. You know that you earn 3% interest on a bank account or other investments. But this could mean two slightly different things:

- *Simple interest* is earned each year by increasing the original amount by 3%.
- *Compound interest* is earned each year by increasing the value of the previous year by 3%.

a) You invest $1000 at 3% interest. Determine the value after 3 years if the interest is calculated as

i) simple interest

ii) compound interest

b) Explain why you think that *simple interest* has the name that it does.

19. A Tax-Free Savings Account (TFSA) allows you to invest money and earn interest without paying any tax on the interest you earn.

a) Imagine you are 19 years old and you put $100 into a TFSA. How much money would you have when you are 60 if you earn 3% compound interest each year? Hint: You may wish to research how to use a spreadsheet to calculate this.

b) If you waited until you were 40 to invest $100, how much would you have at age 60?

Rich Problems

1. Fire code regulations state the maximum number of people allowed to be in rooms and buildings in public spaces. The table shows the minimum allowable floor area per person for different types of room use in British Columbia.

Type of Room or Floor Area	Area Per Person (m^2)
Space with non-fixed seats	0.75
Stages for theatrical performances	0.75
Space with non-fixed seats and tables	0.95
Standing space	0.40
Stadium and grandstands	0.60
Bowling alleys, pool and billiard rooms	9.30
Classrooms	1.85
School shops and vocational rooms	9.30
Reading or writing rooms or lounges	1.85
Laboratories in schools	4.60
Dining and cafeteria space	1.20
Licensed beverage establishment	1.20
Behind kitchen area	9.30

a) Measure the dimensions of the floor of your classroom, or another room in your school or community. Determine the maximum allowable capacity of the room.

b) Imagine that there were ten more people in the room than the maximum allowable. What percent would all of the people in the room be compared with the maximum allowable?

c) If the room had double the number of people allowed, what percent would this represent?

d) Why do you think there are regulations about the maximum allowable capacity of rooms?

2. Estimate the percent of time you spend in a week for various activities outside of school classes.

a) Collect the class results and determine the average for each category. Compare your typical week to the class average. Do you need to add more categories?

b) Create a circle graph showing the results of the survey.

c) Trade graphs with a classmate. Use a protractor to measure each category of each other's graph. Determine the percent of each category.

3. A baseball team has won 50 games out of the 75 played. The team has 45 games still to play. How many of the remaining games must the team win so that it wins 60% of its games for the whole season?

4. Use catalogues or flyers to choose 5 items that add up to exactly $1000. The total cost must include discounts and sales taxes.

Chapter 8 Review

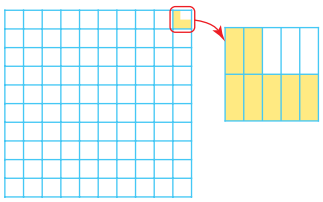
Learning Goals

Inquire and Explore: How are decimals, fractions, and percents interrelated? How can you compare, represent, and communicate two quantities?	
After this section, I can	
8.1	■ visualize, describe, and identify situations where percents ■ are less than 1% ■ are greater than 100% ■ include a decimal or fractional portion
8.2	■ express and make connections among representations of percents, fractions, and decimals, including percents less than 1 and greater than 100, and fractional percents
8.3	■ apply mental math strategies to estimate the percent of a number ■ use multiple approaches to solve problems involving percents less than 1 and greater than 100, including fractional percents
8.4	■ use multiple strategies to solve problems involving combined percents and percent of a percent
8.5	■ use multiple strategies to solve problems involving multiple percents in financial situations

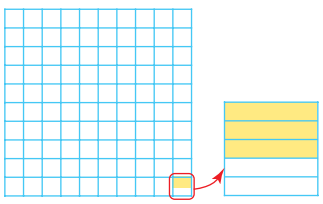
8.1 Understanding Large and Small Percents, pages 256–265

1. What percent does each diagram represent if one completely shaded grid represents 100%?

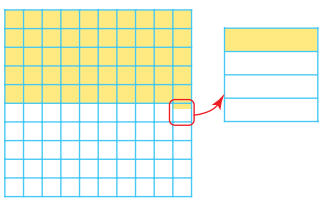
a)



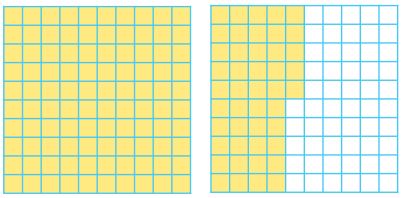
b)



c)



d)



2. Create a visual representation of each percent.

a) 79.1% of students are right-handed.

b) The average person in Canada uses about 223% as much water per day as the average person in France.

c) School enrolment increased by 0.8% this year.

3. Canada's total area is 9 984 670 km^2. Approximately what percent do each of the following countries represent compared with Canada's total area?

a) Russia, the largest country in the world, with a total area of 17 098 242 km^2.

b) Vatican City, the smallest country in the world, with a total area of 0.44 km^2.

8.2 Fractions, Decimals, and Percents, pages 266–273

4. Express each percent as a decimal number and as a fraction.

a) Kyle scored 95.5% on his last test.

b) The store's sales increased by 140%.

c) By getting a car tuned up, you can reduce its emissions by 0.9%.

5. Copy and complete the following table. The first row is done for you.

	Fraction	Decimal	Percent
	$1\frac{2}{5}$	1.4	140%
a)		0.115	
b)			$23\frac{3}{4}$%
c)	$\frac{3}{200}$		
d)		3.85	

6. The triathlon has been included in the summer Olympics since 2000. The race consists of a 1.5-km swim, a 40-km bike ride, and a 10-km run. Canadian Simon Whitfield won the first-ever Olympic gold medal in triathlon in 2000. He also won a silver medal in 2008. His times in this race were 18:45 in the swim, 59:25 on the bike, and 30:48 in the run.

a) What portion of the entire race distance is each component, expressed as a fraction, a decimal, and a percent?

b) What portion of Simon's 2008 race time was each component?

c) Are your answers to parts a) and b) different? Why or why not?

8.3 Percent of a Number, pages 274–279

7. Find each percent using mental math.

a) 115% of 230

b) $80\frac{3}{4}$ % of 50

c) 500% of 0.02

d) $\frac{1}{10}$% of 800

8. Determine each value. Write your answer to the nearest tenth.

a) 137.2% of 28.2

b) 63.8% of 450

c) 0.35% of 1280

d) $\frac{2}{5}$% of 27 000

9. Julia is buying 4 tickets to a concert. The tickets cost $92 each, but there is a 2.5% fee added to the price. What will her total cost be?

8.4 Combined Percents and Percent of a Percent, pages 280–285

10. In a park, 62.5% of the trees are coniferous trees, and 38% of these coniferous trees are Douglas firs. What percent of all the trees in the park are Douglas fir trees?

11. There are 200 people in a building. The number increases by 20% on one day and then decreases by 8% the next day.

a) How many people are there after the two days?

b) What single percent increase or decrease is this equivalent to?

8.5 Percent and Financial Literacy, pages 286–293

12. The cost of an airline ticket is $678 before taxes. There is also 5% GST, 7% PST, 1% airport improvement fee, and 0.75% booking fee. What is the total cost of the ticket?

13. A pair of skates is regularly priced at $289 but is on sale for 20% off. What is the total cost including 5% GST and 7% PST?

14. What is the final sale price at each store? Which is a better buy? Justify your answers mathematically.

Store A: 50% off one day only

Store B: 25% off one day followed by 25% off the reduced price the second day

Connect the Concepts

15. Liam's family is renovating their house. They are changing the sizes of some existing rooms and adding a bedroom. The table shows the areas of rooms before and after the renovation.

Room	Floor area (ft^2)	
	Before Renovation	After Renovation
Total	1930	2450
Kitchen/great room	600	800
Family room	300	300
Hallway/entrance	300	240
Office		100
Master bedroom	240	350
Master bathroom	60	120
Bathroom #2	20	20
Bedroom #2	150	150
Bedroom #3	120	120
Bedroom #4		110
Bathroom #3	90	90
Laundry room	50	50

a) What percent is the total floor area after the renovation compared with before?

b) Use mental math and estimation to determine whether any rooms make up less than 1% of the total floor area after the renovation.

c) Is the kitchen/great room a bigger percent of the house before or after the renovation? Show how you know.

d) After the renovation, the kitchen makes up roughly 45% of the kitchen/great room. What percent of the whole house is this?

e) The total cost of the renovation is estimated to be $20 000 in materials and $15 000 in labour before taxes. If materials are subject to 7% PST and 5% GST but labour is only subject to GST, what is the total cost of the renovation?

f) What percent of the total amount is spent on taxes?

16. In baseball, a player's batting average compares the number of hits to the total number of times at bat. Brendan has been up to bat 80 times so far this season and has 26 hits.

a) What is Brendan's batting average at this point? Express your answer as a fraction, a decimal, and a percent.

b) Batting average is usually shown as a decimal rounded to three places. Do you think a decimal is the best way to express this value? What are some advantages of each of the three ways you represented batting average in part a)?

c) Brendan estimates that 12% of his hits are home runs. What percent of his total times at bat does he get a home run?

d) The record for highest batting average in Brendan's league is 0.368. If he goes up to bat 140 more times this season, how many hits does he need to get in order to beat this record? Justify your reasoning.

e) Is batting average a part-to-part or part-to-whole ratio? Explain.

f) Use your own examples to explain whether a player's batting average can be

i) greater than 100%

ii) less than 1%

iii) a fractional or decimal percent